Problems to prepare for eigenvalues

Note λ is a constant in all these problems.

1. Find the determinant of these matrices. If the determinant is 0, which axis is all the data being squished onto?

   $A = \begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix}$

   $B = \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}$

   $C = \begin{bmatrix} -\lambda & -1 \\ 2 & -\lambda \end{bmatrix}$

2. Find the roots of the following equations (find λ):

   $\lambda^2 + 2 = 0$

   $\lambda^4 - 1 = 0$

3. Compute the matrix-vector multiplications

   $\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} =$

   $\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} =$

   Do you notice anything special about the results?

4. Compute the following derivatives

   $\frac{d}{dt}\left(e^{\lambda t}\right) =$

   $\frac{d}{dx}\left(\ln(x)\right) =$

5. Compute the following integrals (anti-derivatives)

   $\int \frac{dx}{x} =$

   $\int \lambda \, dt =$

6. BONUS:
   Find $x(0.1)$ given $x(0) = 5$ using Euler's method:

   $\frac{dx(t)}{dt} = -x$