

COMBINATORICS

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Ex. 1

HOW MANY WAYS ARE THERE TO PLACE 3 BALLS IN 2 BOXES, EACH CONTAINING 1 BALL MAXIMUM?

(A) (B) (C) \rightarrow $\square \square$

LET'S COUNT:

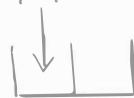
$\square \square$ $\square \square$ $\square \square$ $\square \square$ $\square \square$ $\square \square$ $\square \square$ $\square \square$

WE WILL DISTINGUISH BASED ON THE ORDER IN THIS CASE

WE HAVE SEVERAL COPIES OF EACH BALL

$$S = \{A, B, C\} \quad 3 \text{ ELEMENTS}$$

A, B, C = 3



A, B, C = 3



$$\text{ORDERED COMBINATIONS} = 3 \cdot 3 = 3^2$$

WITH REPLACEMENT

n^r \rightarrow number of boxes

\uparrow
number of elements

Ex. 2

(2)

WHAT IF WE ONLY HAVE 1 COPY OF EACH ELEMENT?

$$A, B, C = 3$$

$$B, C = 2$$



$$3 \cdot 2 = 6$$

GENERAL CASE

$$m \cdot (m-1) \cdot (m-2) \cdots (m-r+1) = m \cdot (m-1) \cdots (m-r+1) \cdot \frac{(m-r)(m-r-1)\cdots 1}{(m-r)(m-r-1)\cdots 1}$$

$$= \frac{m!}{(m-r)!}$$

$$\boxed{r \leq m}$$

ORDERED
COMBINATIONS
WITHOUT REPLACEMENT

IF $r = m \rightarrow$ PERMUTATIONS

Ex. 3

WHAT IF WE DON'T CARE ABOUT THE ORDER?

→ $\boxed{A|B}$
 $\boxed{B|A}$ COUNT AS ONE

UNORDERED
COMBINATIONS
WITHOUT REPLACEMENT

CASE $m=3, r=3$

$\boxed{A|B|C}$ $\boxed{A|C|B}$ $\boxed{B|A|C}$ $\boxed{B|C|A}$ $\boxed{C|A|B}$ $\boxed{C|B|A}$

$$3 \cdot 2 \cdot 1 \rightarrow \text{IN GENERAL} = r(r-1)\cdots 1 = r!$$

$$\frac{m!}{(m-r)!} \rightarrow \frac{m!}{(m-r)! r!} \equiv \binom{m}{r} \equiv \binom{m}{m-r}$$

$\boxed{\text{BINOMIAL COEFFICIENT}}$

WHY IS IT CALLED BINOMIAL?

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BINOMIAL FORMULA

$$(a+b)^m = \sum_{i=0}^m \binom{m}{i} a^{m-i} b^i$$

Ex.

$$(a+b)^4 = (a+b)(a+b)(a+b)(a+b) =$$

- 1 WAY TO COMBINE ALL a s $\rightarrow a^4$
- 4 WAYS TO COMBINE 3 a s AND 1 $b \rightarrow 4a^3b$
- ETC... \hookrightarrow EQUIVALENT TO PLACING 1 " b " BALL IN 4 BOXES

$$= a^4 + b^4 + 4a^3b + 6a^2b^2 + 4ab^3$$

\downarrow

$$\frac{4!}{(4-1)! \cdot 1!} = \frac{4!}{3!} = 4$$