Problem set

1. A group of 25 people participates in a New Year's Eve party. At the stroke of midnight the champagne bottles are uncorked and everyone is exchanging greetings by clinking glasses. How many clinks will you hear in total?

2. What is the probability of drawing a king and a queen consecutively from a deck of 52 cards, without replacement?

3. Let's go back to the problem of the two boxes, one of which contains 8 white balls and 2 black (box A) and the other 2 white and 8 black (box B). You take out a ball from a randomly chosen box and put it back in the other box. This time, we look at the color of the ball, and it is white. Then a ball is extracted from this last box.

    (a) Calculate the conditional probability of obtaining a white ball from box A at the first extraction $P(W_1 \mid A)$.

    (b) Calculate the conditional probability of having extracted a ball from box A given that the extracted ball was white $P(A \mid W_1)$ (*hint: use Bayes' rule*)

    (c) Calculate the probability that a white ball is obtained at the second extraction, given that a white ball was drawn from box A at the first extraction $P(W_2 \mid W_1 \; from \; A)$.

    (d) Calculate the probability of obtaining a white ball at the second extraction. (Remember that we know that at the first extraction we obtained a white ball)

4. We record from five neurons in the motor cortex of a rat when the animal is running. Three of these neurons (excitatory) activate on 90% of the trials but stay silent during the remaining 10%. The other two neurons (inhibitory) are only active on 30% of the trials. We consider five trials recorded from a neuron chosen at random among those five, and we notice that in two trials the neuron was silent and in three trials the neuron was active. Assume that each neuron is independently active or silent during different trials.

    (a) Compute the conditional probability of observing 2 silent trials and 3 active trials knowing that the neuron is excitatory.

    (b) Use Bayes' rule to calculate the probability that the recorded neuron is inhibitory given that in 2 trials the neuron was silent and during 3 trials the neuron was active.

5. Simulate and visualize a binomial random process in python.